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Revisiting Rescheduling: MRP Nervousness and the Bullwhip Effect

Li, Q., and Disney, S.M.

We study the material requirement planning (MRP) system nervousness problem from a dynamic, stochastic and economic perspective in a two-echelon supply chain under first order auto-regressive demand. MRP nervousness is an effect where the future order forecasts, given to suppliers so that they may plan production and organize their affairs, exhibits extreme period-to-period variability. We develop a measure of nervousness that weights future forecast errors geometrically over time. Near-term forecast errors are weighted higher than distant forecast errors. Focusing on replenishment policies for high volume items, we investigate two methods of generating order call-offs and two methods of creating order forecasts. For order call-offs, we consider the traditional order-up-to (OUT) policy and the proportional OUT policy (POUT). For order forecasts, we study both minimum mean square error (MMSE) forecasts of the demand process and MMSE forecasts coupled with a procedure that accounts for the known future influence of the POUT policy. We show that when retailers use the POUT policy and account for its predictable future behavior, they can reduce the bullwhip effect, supply chain inventory costs and the manufacturer's MRP nervousness.

Keywords: Bullwhip Effect, Inventory Management, Material Requirements Planning (MRP), Supply Chain Management, Nervousness

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1. Introduction

It is common practice in automotive, manufacturing, and electronic industries to issue suppliers *call-off orders* (firm orders that must be satisfied immediately), and *order forecasts* (a set of future orders forecasts), [Harrison \(1997\)](#) and [Terwiesch et al., \(2005\)](#). By way of a real example, [Table 1](#) documents a company's orders given to their supplier. In each replenishment cycle (a week in this case, but it could be as short as a few hours, or as long as a month), a firm order—a *call-off order*—is given to the supplier instructing how much to

dispatch now. At the same time, some guidance of the likely, but not guaranteed, orders in the future—the *order forecasts*—are also passed to the supplier. In week 1, the company did not order anything (the zero in the $(1,1)^{\text{th}}$ entry of the matrix in Table 1). However, they forecasted the requirements for the next seven weeks—the $(1, j)^{\text{th}}$ entry of the matrix in Table 1, $j = \{2, 3, \dots, 8\}$. This future guidance together with inventory, production, and delivery information is used by the supplier to initiate production, procure materials, schedule labor, and plan capacity acquisitions. In practice, material requirements planning (MRP) systems are used to facilitate this task.

In the second week, a new call-off quantity (the $(2,2)^{\text{th}}$ entry of the matrix in Table 1) and seven new future order forecasts were generated. The call-off order was the same as the forecast for the week 2 made in week 1. However, the order forecasts for weeks 3, 4, 6 and 7 were updated. In week 3, the call-off for week 3 was no longer 5940 as predicted in the previous week, but 5400, and the week 4 forecast changed from 7020 to 5940.

Quantity		For delivery in week									
		1	2	3	4	5	6	7	8	9	10
Orders placed in week	1	0	1620	2160	4860	8100	8100	8100	0	-	-
	2	-	1620	5940	7020	8100	7560	4860	7020	0	-
	3	-	-	5400	5940	8100	7560	4860	7020	0	7020

Table 1. Real-life example of system nervousness (Key: Call-off orders in bold)

When new, and presumably more accurate data becomes available regarding future requirements, the supplier's previously calculated schedule needs revision. Order forecasts that change every period make the supplier's MRP system *nervous* (Mather 1977). Nervousness is undesirable as quantity increases within the lead-time cannot be met without expediting production or delivery. Alternatively, volume decreases result in excessive inventory accumulating. Nervousness also leads to reduced productivity and confusion on the shop floor. The variability of the call-off order leads to the bullwhip problem (Wang and

Disney 2016). Bullwhip results in production and replenishment plans that have unduly high variability causing capacity losses and increased inventory requirements (Disney and Lambrecht 2008). Distressingly, many companies will experience both systems nervousness and bullwhip simultaneously.

While suppliers may not be able to measure and transfer the cost of nervousness to their customers directly, all costs—including the cost of nervousness—must be absorbed into the supply chain somewhere. Therefore, we focus on how one might produce more reliable future guidance so as to reduce system nervousness. Recently dynamic supply chain studies have ignored the nervousness problem as it is difficult to quantify the cost of nervousness directly. Indeed, there does not appear to be an established cost function for nervousness in the literature (although Ho (2005) provides a review of nervousness definitions). However, the impact of nervousness upon inventory and capacity (bullwhip) costs can be readily measured and do have established cost functions. Inventory and bullwhip measures are important as companies are not only concerned about their costs but also their suppliers, especially when the same company owns an assembly site and its component supplier.

MRP/enterprise resource planning (ERP) systems are often used in industry to plan and record commercial supply chain transactions, even in lean production environments. The workflow in MRP/ERP systems typically starts by updating forecasts for final assembly items by delivery location over the next 3-6 months in (often weekly) time buckets. Demand and forecast information passes to a planning book where, together with inventory, production and delivery information, a master production schedule (MPS) is calculated. Two approaches are usually used to construct the MPS. The first method, suitable for high volume products produced every period, is the order-up-to (OUT) policy. The second method is the economic order quantity method, used for low volume products, or those with high set-up/changeover costs. Then the MPS is exploded out down through the bill of materials (BOM) and work

assigned to individual production facilities (machines, workstations, assembly lines, etc.) in a detailed production planning module that considers changeover costs and times. Here minute-by-minute, hour-by-hour production plans can often be manipulated in a Gantt chart like interface to account for factors such as maintenance, urgent shipping requirements, raw material availability, and forthcoming holidays/factory shutdowns. Finally, the MRP system issues order call-off and order forecasts to suppliers. This task requires accurate BOM, routing, and lead-time information.

This paper considers a generalization to the OUT policy, called the *proportional order-up-to* (POUT) policy, based on adding a proportional controller into the inventory position feedback loop of the OUT policy (Disney and Towill 2003). We investigate how the POUT policy reduces nervousness when compared to the OUT policy. Furthermore, we also propose a mechanism called *proportional future guidance* (PFG). The PFG mechanism constructs order forecasts as a sum of demand forecasts and the predictable future consequences of the proportional feedback controller. We will show the PFG mechanism further reduces MRP nervousness. Our policy is easy to understand, simple to use, and provides competitive performance without the need for sophisticated IT systems to share demand information.

We focus on the OUT policy type of MPS calculation for the following reasons. Principally the OUT policy (and its POUT variant) is a linear system. Hence we can fully characterize the solution. Second, the OUT policy is suitable for high volume items which typically use the largest proportion of available capacity. Third, the POUT policy is known to be able to smooth the call-off orders and reduce capacity costs. Finally, the POUT/PFG policy is practically implementable as we have incorporated it into a global manufacturer's ERP system by custom coding user defined macros in the ERP planning book (see Disney et al., (2013) for an early report on this project). We have also worked with companies to

implement the POUT policy by custom coding turnkey IT systems and developing Excel-based decision support systems (for example, see [Potter and Disney \(2010\)](#)).

We take a descriptive, rather than a prescriptive, stance to this research. That is, we are interested in understanding how real, implementable policies behave rather than finding optimal policies for a given cost function. Furthermore, to gain analytical insights we have assumed a linear system exists. Importantly, we assume that the manufacturer guarantees supply to the retailer. If the manufacturer does not have enough stock to fill an order, he obtains the backlogged quantity from an alternative source with the same lead-time. He is also responsible for resupplying this source later at a penalty cost. This assumption is quite common in the literature (see [Lee et al., \(2000\)](#)), and allows us to fully characterize the policies studied, a feat not often achievable with non-linear models. Further assumptions in our model include known and constant lead-times, no capacity constraints, the free return of excess inventory, and no quality losses or unreliable supply.

Our contribution is to show that the POUT policy, known to be able to reduce the bullwhip effect, is also able to reduce MRP nervousness even without taking particular account of its anticipated impact on future orders. Furthermore, if we do account for the POUT policy's future impact, nervousness is reduced even further. Our paper is organized as follows: §2 reviews the literature. §3 defines the model setup. §4 measures the nervousness induced by the proposed policies. §5 investigates economic performance via capacity and inventory costs. §6 provides managerial implications, and §7 concludes. Appendix I lists notation and Appendices II-IV provide mathematical derivations.

2. Literature review

Frequently changing schedules is the essence of the MRP nervousness problem ([Mather 1977](#)). Frequently changing schedules lead to reduced productivity, increased costs, reduced

inventory availability, and confusion on the shop floor (Hayes and Clark 1985). A range of solutions has been proposed to reduce nervousness, which we now review.

Mather (1977) proposed filtering insignificant rescheduling messages to avoid disruptions to open orders (open order are orders that have been placed, but not yet received). Significant rescheduling can be accommodated by revising the due dates of open orders. Ho (2005) argued the effectiveness of this approach depends on the operating environment. Carlson et al., (1979) proposed an objective function that included a changeover cost that depends on the previous schedule. The cost for a previously scheduled changeover is penalized less than a previously unscheduled one. The objective function ensures schedules are generated that balance the costs of nervousness and responsiveness. Additionally, lot-sizing algorithms have been found to have a significant impact on both nervousness (Ho and Ireland 1998) and costs (de Bodt and Van Wassenhove 1983).

Zhao et al., (1995) and Kadipasaoglu and Sridharan (1995) studied order- and period-based freezing methods, finding that freezing methods can reduce nervousness. Freezing is also considered as the most effective strategy to reduce nervousness in multi-level MRP systems (Sahin et al., 2013). However, the act of freezing was deemed to be a potential source of nervousness itself. Tang and Grubbström (2002) found that forecast errors have an impact on the re-planning frequency and the length of the frozen period, particularly when the safety stock is not optimal. Xie et al., (2003) revealed that the interaction between freezing parameters and forecast errors significantly affects the performance of a manufacturing system.

Buffers—of stock, capacity, or lead-time—can be used to reduce nervousness. Safety stock and safety capacity are more economical than rescheduling (Schmitt 1984). Grasso and Taylor (1984) assessed the impact of safety stock and safety lead-time on cost, finding that

safety stock is more efficient than safety lead-time. [Sridharan and LaForge \(1989\)](#) indicated that a small amount of safety stock improved schedule stability but the act of increasing safety stock may itself induce instability. [Grubbström and Molinder \(1996\)](#) calculated optimal safety stock and production plans under Poisson distributed demand.

Recent studies on MRP and MPS have turned to programming approaches to improve industrial applications. [Herrera et al., \(2016\)](#) proposed a reactive approach based on parametric mixed-integer programming for manufacturers to reduce nervousness. [Rossi et al., \(2016\)](#) combined the traditional MRP procedure with an approach based on linear programming to consider capacity constraints. Integrating a stochastic programming model into a hierarchical production planning and control system, [Englberger et al., \(2016\)](#) created a smoothed MPS at the expense of increased safety stocks.

While the literature has mainly investigated nervousness from a single-stage perspective, some research has considered multi-echelon scenarios where the future guidance is also established. [Sahin et al., \(2008\)](#) evaluated the interaction between the MPS and the future guidance via cost and schedule performance at both a manufacturer and a vendor. They found that the manufacturer's optimal MPS policy often sabotages the vendor's cost performance. [Robinson et al., \(2008\)](#) considered interaction effects in a two-stage scenario focusing on the schedule flexibility after the frozen period. Their results suggest channel flexibility can yield substantial costs savings. [Nedaei and Mahlooji \(2014\)](#) extended this approach to consider smoothness-based objectives in a two-echelon supply chain. [Sahin et al., \(2013\)](#) provided a comprehensive review of rolling horizon planning studies.

A few studies have also considered the nervousness and bullwhip problems together. Integrated control policies were applied Phillips' supply chain by [de Kok et al., \(2005\)](#). An ERP system created synchronized work-orders across the entire chain, reducing both the

bullwhip and nervousness problems. [Chen and Lee \(2009\)](#) investigated a general demand pattern and linear, generalized order-up-to replenishment policies based on weighted future order forecasts. They showed how weighted forecasts might be used to reduce order variability, order uncertainty, and inventory costs. [Bray and Mendelson \(2015\)](#) extended this idea by arguing that firms amplify last minute surprises more than the forecasts in the distant future¹.

3. Methods to generate call-off orders and order forecasts

We investigate a two-echelon supply chain with a single retailer and a single manufacturer. Either player could be a retailer, distributor, manufacturer, or supplier, but we use the term retailer (manufacturer) as it is easily recognized as the downstream (upstream) player. We assume both players are motivated to minimize capacity and inventory related costs. We have worked with a global company who owned and operated both the downstream assembly site and the upstream component manufacturer. He was concerned with capacity and inventory costs in both echelons. We have also worked with a UK grocery retailer who was concerned with in-store availability as well as capacity and inventory related costs in their distribution centers. The capacity costs were associated with variable workloads in the cross-docking operations as well as the inefficient use of transportation.

We consider the following setting. The retailer satisfies end consumer demand from stock replenished after a lead-time of T_p time periods. The retailer's order must be dispatched immediately from the manufacturer's finished goods inventory. As such, the manufacturer does not get any frozen period. The manufacturer also experiences a lead-time

¹ This stream of research has used the moniker *order uncertainty* to describe the nervousness effect.

We have elected not to do this to avoid confusion with the risk/uncertainty framework. Furthermore, using the nervousness term draws attention to the established literature in the MRP field.

(of T_s) to replenish his finished goods inventory. Thus he must also operate a make-to-stock system. We assume that any changeover costs are either non-existent (i.e. there are no changeovers) or constant and independent of the call-off quantities.

3.1. Call-off order and order forecast notation

Let $\{O_t, \hat{O}_{t,t+1}, \hat{O}_{t,t+2}, \dots, \hat{O}_{t,t+j}\}$ be the set of information passed to the manufacturer by the retailer each period: O_t , is the call-off order given to the manufacturer at time t ; $\hat{O}_{t,t+j}$ is the prediction made at time t of the order quantity in the period $t+j$, O_{t+j} . $j \in \mathbb{N}^+$, where the number of future predictions provided is linked to the length of the manufacturer's planning horizon. Practically, in a weekly planning system, organizations usually prepare forecasts for up to 13 or 26 weeks in advance. Often order forecasts near the horizon are aggregated into monthly buckets.

3.2. Scenario A: The proportional order-up-to policy with minimum mean squared error forecasts

In Scenario A, the retailer generates call-off orders via the POUT policy with

$$O_t = \hat{D}_{t,t+T_p+1} + \frac{1}{T_i} \left(TNS - RNS_t + \sum_{i=1}^{T_p} (\hat{D}_{t,t+i} - O_{t-i}) \right), \quad (1)$$

where O_t is the order placed at time t that must be dispatched in this period. $T_p \in \mathbb{N}^0$ is the replenishment lead-time. $\hat{D}_{t,t+T_p+1}$ is a forecast of demand, made at time t of demand T_p+1 periods ahead. TNS is the target net stock, a time-invariant safety stock that can be set to minimize inventory holding and backlog costs via the newsvendor principle (Brown 1962). RNS_t is the retailer's net stock at time t . A positive (negative) net stock denotes inventory holding (backlogs). The final component of (1) is a sum over the lead-time, T_p , of the

difference between forecasted demand over the lead-time, $\sum_{i=1}^{T_p} \hat{D}_{t,t+i}$, and the call-off orders placed, but not yet received, $\sum_{i=1}^{T_p} O_{t-i}$.

T_i is the proportional feedback controller. $0.5 < T_i < \infty$ is required for stability (Disney 2008). The POUT policy (1) allows access to a wide range of replenishment strategies. For example, (1) degenerates into the OUT policy when $T_i = 1$. The POUT policy can reduce the bullwhip effect in the supply chain when $T_i > 1$. For i.i.d. demand with MMSE forecasting, $\sigma_O^2 = \sigma_D^2 / (2T_i - 1)$, Disney et al., (2004). For arbitrary demand when $T_i = \infty$ the variance of the orders equals the variance of the forecasted demand, $\sigma_O^2 = \sigma_D^2$. If the demand forecasts are constant (say for all t and x , $\hat{D}_{t,t+x} = \mu$, where μ is the average demand), and $T_i = \infty$, then $\sigma_O^2 = \sigma_D^2 = 0$ and a *level scheduling* strategy exists. When $\hat{D}_{t,t+x} = \mu$ and $T_i = 1$ then $O_t = D_t$ and a *pass-on-orders* strategy exists.

The inventory balance equation, $RNS_t = RNS_{t-1} + O_{t-T_p-1} - D_t$, completes the system. The retailer uses minimum mean squared error (MMSE) demand forecasts for the future order forecasts,

$$\hat{O}_{t,t+j} = \hat{D}_{t,t+T_p+1+j}, \quad j \in \mathbb{N}^+. \quad (2)$$

We denote this approach as the POUT/MMSE strategy and use the subscript A to refer to this strategy in equations. We use the term OUT/MMSE (and the subscript A,1) to describe Scenario A when $T_i = 1$.

3.4. Scenario B: The proportional order-up-to policy with proportional future guidance

In Scenario B, (1) generates the call-off orders. However, the future guidance is based on,

$$\hat{O}_{t,t+j} = \hat{D}_{t,t+T_p+1+j} + \frac{1}{T_i} \left(\frac{T_i-1}{T_i} \right)^j \left(TNS - RNS_t + \sum_{i=1}^{T_p} \left(\hat{D}_{t,t+i} - O_{t-i} \right) \right), \quad T_i \neq 1. \quad (3)$$

This method accounts for the predictable future consequences of the POUT policy reacting to the current error in the inventory position. We denote this approach the POUT/PFG strategy and use the subscript B to refer to this policy in equations.

4. Measuring systems nervousness

We assume that the manufacturer reschedules his production quantity each period to reflect the latest available information, hence he has no frozen period. The variance of the j -step ahead order forecast error is

$$\Delta[j] = \text{var}(\hat{O}_{t-j,t} - O_t). \quad (4)$$

$\Delta[j]$ is an increasing function of j . This variance is a measure of the j period ahead order forecast provided by the retailer. Note, we are unconcerned whether the forecast error is positive or negative. As a forecast error in the near future is more costly (or at least harder to deal with) than one in the distant future we adopt a geometrically weighted sum of order forecast error variances² as a measure of nervousness:

$$\Delta = \sum_{j=1}^{\infty} w(1-w)^{j-1} \Delta[j]. \quad (5)$$

² It is widely accepted in the literature that rescheduling an open order in the near future is more costly than one in the distant future. It was reflected in the change cost procedure by Carlson et al. (1979). The methods to evaluate nervousness from Sridharan et al. (1988), Kimms (1998), Pujawan (2004) and Kabak and Ornek (2009) also considered that there is either no consequences of distant change, an equal weight, or a proportional weight. These measures are only amenable to numerical analysis. However, our geometrically weighted forecast error results in a closed form solution, a desirable property in a mathematical modelling study.

Here the geometric weighting factor, w , determines how quickly the variability of the future order forecast errors decay away (in much the same way as the forecasting parameter in exponential smoothing behaves). When w is large, the influence of the forecast error decays away more quickly than when w is small. When $0 < w < 1$, the sum in (5) converges to a finite number, allowing us to compare the nervousness produced by different replenishment system designs. For a given w , smaller Δ indicate more accurate future guidance, (that is, less nervousness is created), and the easier it will be for companies to organize their activities to meet demand. w should be selected to reflect the period over which the future order forecasts are relevant. For example, w near zero is chosen when the forecasts errors over an extended forecast horizon are important (perhaps because of a long lead-time) and w near one is selected when only the one period forecast error is important.

We assume the retailer faces a first order auto-regressive random demand, AR(1) (Box et al., 1994). The AR(1) process was selected as it is the simplest demand process without a constant future forecast. The AR(1) demand process was also found to be representative of: over 80 electronic products by Lee et al., (2000), and grocery demand in Hosoda et al., (2008). Disney et al., (2016) find that the i.i.d. demand pattern (a particular case of the AR(1) process) was representative of the demand for industrial printers. The mean centered AR(1) process is given by

$$D_t = \mu + \rho(D_{t-1} - \mu) + \varepsilon_t, \quad (6)$$

where, D_t is the demand in period t , and ρ is the auto-regressive parameter. $-1 < \rho < 1$ ensures a stable and invertible demand process (Box et al., 1994). The error term, ε_t , is a white noise random process with zero mean and a variance of σ_ε^2 .

We will now derive variance expressions that hold when ε_t is drawn from any continuous distribution. We may assume that the average demand $\mu = 0$ when we investigate

variances (but $\mu > 0$ will be needed in our economic analysis later). Box et al., (1994) show the variance of AR(1) demand is

$$\sigma_D^2 = \sigma_\varepsilon^2 / (1 - \rho^2). \quad (7)$$

The OUT (and POUT) policy requires two MMSE forecasts of the AR(1) demand:

$$\hat{D}_{t,t+T_p+1} = \rho^{T_p+1} D_t, \text{ and} \quad (8)$$

$$\sum_{i=1}^{T_p} \hat{D}_{t,t+i} = \sum_{i=1}^{T_p} \rho^i D_t = D_t \rho (1 - \rho^{T_p}) / (1 - \rho). \quad (9)$$

Appendix I shows that the nervousness of the POUT/MMSE strategy is given by

$$\Delta_A = \sigma_\varepsilon^2 \left(\frac{\kappa^2}{2T_i - 1} + \frac{2\kappa\rho^{T_p+1}}{T_i - (T_i - 1)(1 - w)\rho} + \frac{\rho^{2T_p+2}}{1 - (1 - w)\rho^2} \right), \quad (10)$$

and that the nervousness of the POUT/PFG strategy is:

$$\Delta_B = \sigma_\varepsilon^2 \left(\frac{\kappa^2}{2T_i - 1 + w(T_i - 1)^2} + \frac{2\kappa\rho^{T_p+1}}{T_i - (T_i - 1)(1 - w)\rho} + \frac{\rho^{2T_p+2}}{1 - (1 - w)\rho^2} \right), \quad (11)$$

where $\kappa = (1 - \rho^{T_p+1}) / (1 - \rho)$. The behavior of κ is rather complex and hints at an odd-even lead-time effect that we will see throughout our analysis. When $\rho > 0$, κ is increasing in T_p and ρ . When $\rho < 0$ and the lead time T_p is even (odd), it is convex (increasing) in ρ . Both Δ_A and Δ_B tend to $\Delta = \sigma_\varepsilon^2 \rho^{2(T_p+1)} / (1 - (1 - w)\rho^2)$ as $T_i \rightarrow \infty$.

By setting $T_i = 1$ in (10) we obtain the nervousness expression for the OUT/MMSE policy:

$$\Delta_{A,1} = \sigma_\varepsilon^2 \left(\kappa^2 + 2\kappa\rho^{T_p+1} + \frac{\rho^{2T_p+2}}{1 - (1 - w)\rho^2} \right). \quad (12)$$

As $\lim_{\rho \rightarrow 1} \Delta_{A,1} = \infty$ and $d\Delta_{A,1}/d\rho > 0$ when $0 \leq \rho < 1$, the nervousness induced by the OUT/MMSE strategy, $\Delta_{A,1}$, is increasing in ρ for non-negatively correlated demand. The

derivative at $\rho \rightarrow -1$ is negative and $\Delta_{A,1}|_{\rho \rightarrow -1} < \Delta_{A,1}|_{\rho=0}$. Therefore $\Delta_{A,1}$ must have at least one minimum³ between $-1 < \rho < 0$. (12) has a lower bound of

$$\lim_{w \rightarrow 1} \Delta_{A,1} = \sigma_\varepsilon^2 \left(\kappa + \rho^{T_p+1} \right)^2 = \Delta_{A,1}[1], \quad (13)$$

and an upper bound of

$$\lim_{w \rightarrow 0} \Delta_{A,1} = \sigma_\varepsilon^2 \left(\kappa^2 + 2\kappa\rho^{T_p+1} + \frac{\rho^{2T_p+2}}{1-\rho^2} \right). \quad (14)$$

Note (13) also represents the variance of the one-period-ahead order forecast error, $\Delta[1]$.

Proposition 1: In the majority of cases, for positively correlated demand, the nervousness induced by the OUT/MMSE policy is larger than the variance of the end customer demand.

Proof: The minimum nervousness produced by OUT/MMSE can be derived from the lower bound (13) as

$$\Delta_{A,1} = \sigma_\varepsilon^2 \left(\frac{1-\rho^{T_p+2}}{1-\rho} \right)^2 = \sigma_D^2 (1-\rho^2) \left(\sum_{i=0}^{T_p+1} \rho^i \right)^2. \quad (15)$$

Only when ρ negative or near $\rho=1$, does $\Delta_{A,1} < \sigma_D^2$ hold. In the region where most real demand patterns occur (that is, $0 < \rho < 0.7$, see [Lee et al., 2000](#)), $\Delta_{A,1} > \sigma_D^2$. Therefore, for all T_p and w , $\Delta_{A,1}$ is likely to be greater than σ_D^2 . \square

4.1. The impact of the proportional feedback controller on nervousness

In this section, we study the impact of the proportional feedback controller, T_i , on nervousness.

³ Extensive numerical investigations suggest that only one minimum exists, but we remain unable to formally prove this.

Proposition 2: In a two-echelon supply chain facing i.i.d. demand, with $T_i > 1$, the proportional feedback controller can reduce the manufacturer's nervousness. The POUT/PFG strategy creates the least nervous system.

Proof: When $\rho = 0$, (12), (10) and (11) reduces to

$$\Delta_{A,1} = \sigma_\varepsilon^2, \quad (16)$$

$$\Delta_A = \sigma_\varepsilon^2 / (2T_i - 1), \text{ and} \quad (17)$$

$$\Delta_B = \sigma_\varepsilon^2 / (2T_i - 1 + (T_i - 1)^2 w), \quad (18)$$

respectively. We observe that nervousness is independent of the lead-time when there is no demand correlation. Both (17) and (18) are decreasing in T_i . When $T_i > 1$ in (17) and (18), $\Delta_{A,1} > \Delta_A > \Delta_B$. \square

Proposition 3: For positively correlated demand, nervousness decreases in T_i .

Proof: Observe (12) and (10); the coefficient of the first addend, κ^2 , changes from 1 to $1/(2T_i - 1)$; the coefficient of the second addend, $2\kappa\rho^{T_p+1}$, changes to $(T_i - \rho(T_i - 1)(1 - w))^{-1}$; the third addend remains the same. When $T_i > 1$, $2T_i - 1 > 1$ and $T_i - (T_i - 1)(1 - w)\rho > 1$, then, for $\rho > 0$,

$$\kappa^2 > \frac{\kappa^2}{2T_i - 1}, \text{ and} \quad (19)$$

$$2\kappa\rho^{T_p+1} > \frac{2\kappa\rho^{T_p+1}}{T_i - (T_i - 1)(1 - w)\rho}. \quad (20)$$

Thus, $\Delta_B < \Delta_A$ when $\rho > 0$ and $T_i > 1$.

The first derivative,

$$\partial_{T_i} \Delta_A = \left(-\frac{2\kappa^2}{(2T_i - 1)^2} - \frac{2\kappa\rho^{T_p+1}(1 - (1 - w)\rho)}{(T_i - (T_i - 1)(1 - w)\rho)^2} \right) \sigma_\varepsilon^2 < 0 \quad (21)$$

when $\rho \geq 0$. Thus Δ_A is strictly decreasing in $T_i \in (0.5, \infty)$. \square

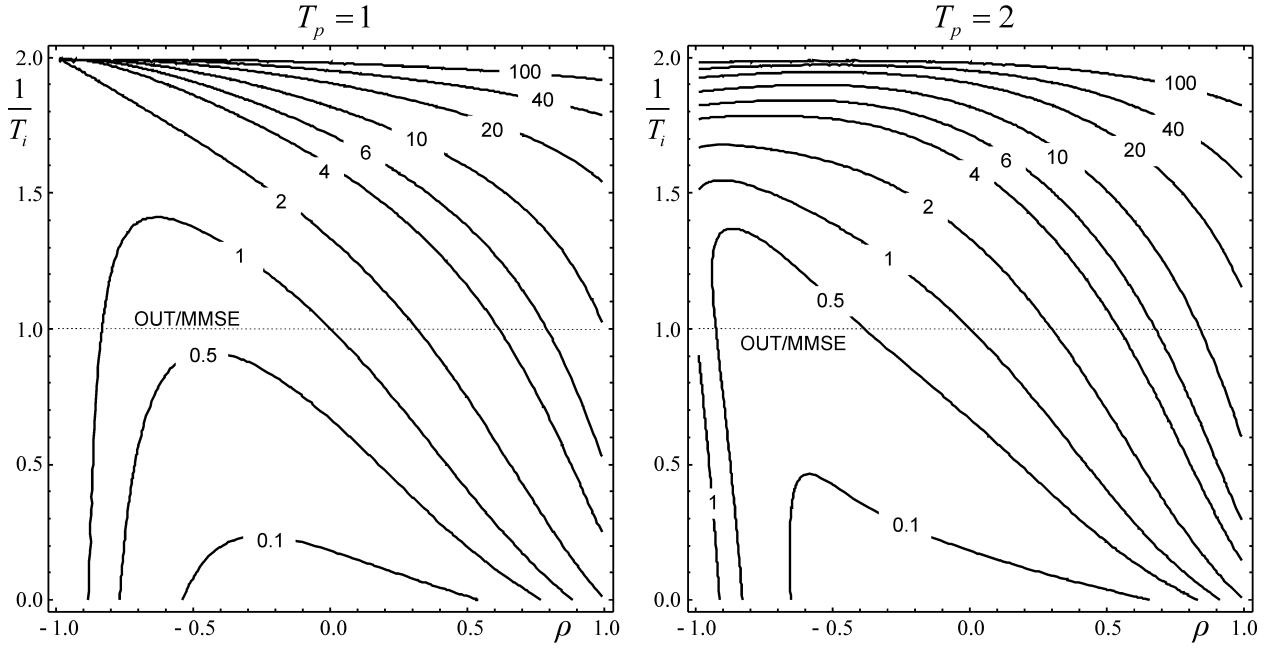


Figure 1. Nervousness in the POUT/MMSE policy when $w=0.5$ under AR(1) demand

Figure 1 illustrates Δ_A for various T_i , ρ , and T_p , when $w=0.5$ and $\sigma_\varepsilon^2=1$. Using $1/T_i$ in the ordinate allows us to plot the entire stability region of T_i . If $\rho > 0$, T_i significantly reduces nervousness. Future guidance becomes more accurate as T_i increases.

For negative ρ , (19) still holds. However, (20) only holds when the lead-time T_p is an odd number. Thus, an odd T_p and $T_i > 1$ is a sufficient condition for $\Delta_A < \Delta_{A,1}$. Proposition 1 and 2 reveal that $T_i > 1$ and $\rho \geq 0$ is another sufficient condition for $\Delta_A < \Delta_{A,1}$. If T_p is even,

$$\lim_{\rho \rightarrow -1} \Delta_{A,1} = \frac{1}{w} - 1, \text{ and} \quad (22)$$

$$\lim_{\rho \rightarrow -1} \Delta_A = \frac{1}{w} + \frac{1}{2T_i - 1} + \frac{2}{1 - T_i(2 - w) - w}. \quad (23)$$

(22) is greater than (23), when $(2-w)^{-1} < T_i < 1$ and $0 < w < 1$. This shows that a nervousness reduction is possible for an even lead-time, but that a $T_i < 1$ may be required for negatively correlated demand. Figure 1 confirms that increasing T_i under an even lead-time might amplify nervousness when demand is highly negatively correlated. Although the lead-time is an exogenous variable in many situations, it can be strategically changed. For example, alternative production technologies or transport modes may alter lead-times significantly. Appropriate supplier selection or contract terms may also allow one to modify or specify a lead-time. Perhaps, if the lead-time cannot be reduced, it could be artificially increased.

4.2. The impact of the proportional future guidance policy

In this section, we study the consequences of using the PFG policy to generate order forecasts.

Proposition 4: For $\rho=0$, the POUT/PFG strategy exhibits less nervousness, particularly in the near future, than the POUT/MMSE strategy.

Proof: For $\rho=0$, the variance of the j period ahead order forecast error in Scenario A and B are

$$\Delta_A[j] = \sigma_\varepsilon^2 \sum_{n=0}^{\infty} \left(T_i^{-1} \left((T_i - 1) T_i^{-1} \right)^n \right)^2 = \sigma_\varepsilon^2 / (2T_i - 1), \quad (24)$$

and

$$\Delta_B[j] = \sigma_\varepsilon^2 \sum_{n=0}^{j-1} \left(T_i^{-1} \left((T_i - 1) T_i^{-1} \right)^n \right)^2 = \sigma_\varepsilon^2 \left(\frac{1 - \left((T_i - 1) T_i^{-1} \right)^{2j}}{2T_i - 1} \right). \quad (25)$$

(24) is decreasing in T_i and independent of j and T_p . $T_i > 0.5$ ensures stability. (25) decreases in T_i and increases in j . As $j \rightarrow \infty$, (25) converges to (24). Therefore, $\Delta_A[j] - \Delta_B[j] > 0$ and decreases in j . \square

Proposition 4 indicates, and Figure 2 verifies, that the POUT/PFG strategy can reduce nervousness in the near future; presumably this is of greater practical benefit than an equivalent nervousness reduction in the distant future.

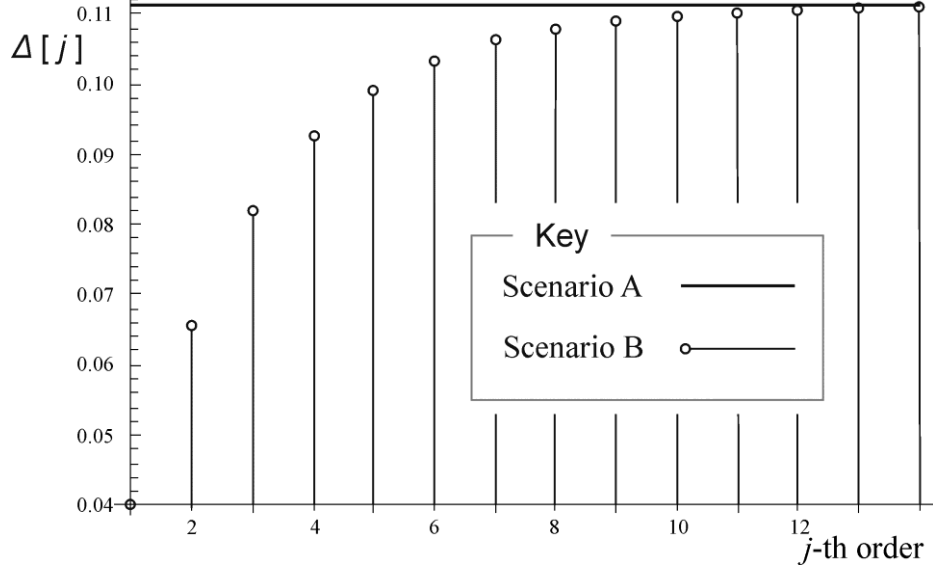


Figure 2. The accuracy of forecasted orders for i.i.d. demand when $T_i = 5$ and $\sigma_\varepsilon^2 = 1$

Proposition 5: For any ρ , the POUT/PFG strategy is less nervous than the POUT/MMSE strategy.

Proof: Observe, from (10) and (11), the difference between the POUT/MMSE and POUT/PFG strategies nervousness is

$$\Delta_A - \Delta_B = \left(\frac{w(T_i - 1)^2 \kappa^2}{(2T_i - 1)(2T_i - 1 + w(T_i - 1)^2)} \right) \sigma_\varepsilon^2. \quad (26)$$

Eq. (26) is always positive when $T_i \in (0.5, 1) \cup (1, \infty)$ implying that $\Delta_B < \Delta_A$. \square

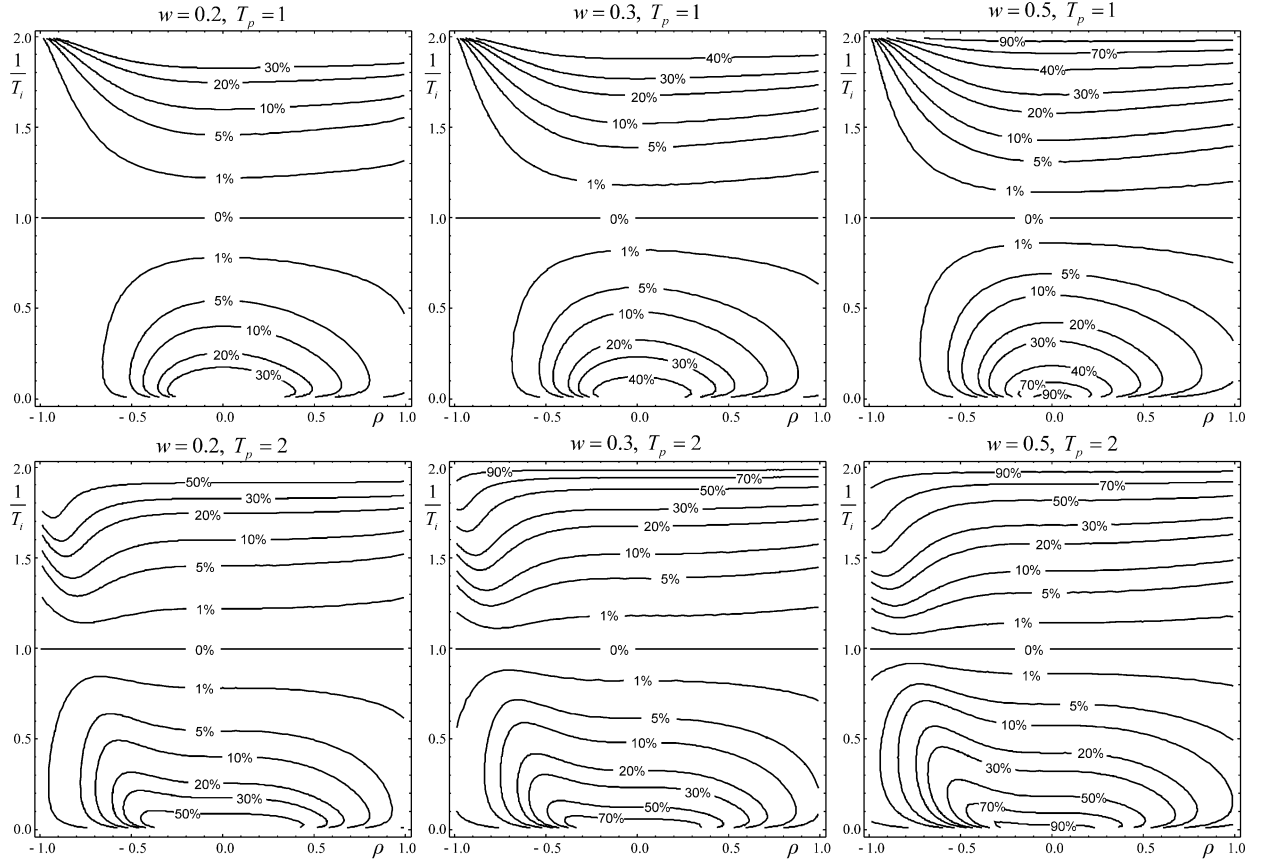


Figure 3. Percentage improvements in nervousness resulting from changing the POUT/MMSE to the POUT/PFG policy

Figure 3 illustrates the percentage accuracy improvement, $((\Delta_A - \Delta_B)/\Delta_A) \times 100\%$, in six numerical settings. It shows that the PFG strategy can further reduce nervousness compared to the MMSE strategies. These improvements reduce as $\rho \rightarrow 1$. When $T_i \rightarrow 0.5$ or when $T_i \rightarrow \infty$ more accurate future guidance exists. Lead-times also have a strong influence on nervousness reduction. The odd-even lead-time effect in both (10) and (11), can be seen in Figure 3. Longer lead times result in larger reductions in nervousness from the POUT/PFG policy. Even lead-times remain undesirable under the POUT/PFG policy with highly negative demand correlation.

In summary, the POUT/PFG policy reduces nervousness compared to the POUT/MMSE policy. For non-negative demand correlation, nervousness reduction is

guaranteed when $T_i > 1$. If demand is strongly negatively correlated, a careful choice of T_p and T_i is required. Odd lead-times allow a form of temporal aggregation as the odd lead-time and the review period mean that an even number of negatively correlated demands are aggregated into the order-up-to level and the inventory position. This pooling effect reduces the variability in the system.

5. Capacity and inventory performance

In this section, we extend our model and analysis to include an upstream manufacturer. We assume that the manufacturer has to dispatch his customer order within the current period, but that he has a lead-time of T_s . The cooperative and trusting manufacturer incorporates the retailer's future guidance into his planning process by setting his production orders to

$$P_t = \left(\hat{O}_{t,t+T_s+1} + \sum_{j=1}^{T_s} \hat{O}_{t,t+j} \right) - \left(\hat{O}_{t-1,t+T_s} + \sum_{j=1}^{T_s} \hat{O}_{t-1,t-1+j} \right) + O_t. \quad (27)$$

Notice that (27) is an OUT policy. In (27), P_t is the production order quantity made by the manufacturer at time t . The demand that the manufacturer receives at time t is the retailer's order, O_t . The forecasts for the demand during $\left(\sum_{j=1}^{T_s} \hat{O}_{t,t+j} \right)$, and in the period after $\left(\hat{O}_{t,t+T_s+1} \right)$, the manufacturer's lead-time, T_s , are drawn from the retailer's future guidance. The finished goods inventory maintained by the manufacturer (the manufacturer's net stock, MNS_t) is governed by

$$MNS_t = MNS_{t-1} + P_{t-T_s-1} - O_t. \quad (28)$$

5.1. Inventory variance in three scenarios

The retailer's inventory variance under the POUT policy is

$$\sigma_{RNS_A}^2 = \sigma_{RNS_B}^2 = \frac{\sigma_\varepsilon^2}{(1-\rho)^2} \left(1 + T_p + \frac{\rho(1-\rho^{T_p+1})(\rho^{T_p+2} - \rho - 2)}{(1-\rho^2)} + \left[\frac{\left(\frac{T_i-1}{T_i}\right)^2 (1-\rho^{T_p+1})^2}{\left(1 + \frac{T_i-1}{T_i}\right) \frac{1}{T_i}} \right] \right), \quad (29)$$

which has a minimum at $T_i = 1$ (Hosoda and Disney 2006). When $T_i = 1$, (29) represents the retailer's net stock variance under the OUT policy (the term in the square brackets disappears).

Appendix III shows the manufacturer's net stock variance in Scenarios A and B are,

$$\sigma_{MNS_A}^2 = \sigma_\varepsilon^2 \left(\sum_{n=0}^{T_s} \left(\sum_{i=0}^n \left(\rho^{T_p+1+i} + \frac{1-\rho^{T_p+1}}{T_i(1-\rho)} \left(\frac{T_i-1}{T_i} \right)^i \right) \right)^2 + \sum_{n=0}^{\infty} \left(\left(\frac{T_i-1}{T_i} \right)^{1+n} \left(1 - \left(\frac{T_i-1}{T_i} \right)^{T_s+1} \right) \frac{1-\rho^{T_p+1}}{1-\rho} \right)^2 \right), \quad (30)$$

and

$$\sigma_{MNS_B}^2 = \sigma_\varepsilon^2 \sum_{n=0}^{T_s} \left(\sum_{i=0}^n \left(\rho^{T_p+1+i} + \frac{1-\rho^{T_p+1}}{T_i(1-\rho)} \left(\frac{T_i-1}{T_i} \right)^i \right) \right)^2 = \frac{\sigma_\varepsilon^2}{(1-\rho)^2} \sum_{n=0}^{T_s} \left(\left(\frac{T_i-1}{T_i} \right)^{1+n} (\rho^{1+T_p} - 1) + 1 - \rho^{2+n+T_p} \right)^2. \quad (31)$$

Both (30) and (31) have closed forms. However, we leave them unevaluated as they result in very long expressions. Figure 4 shows that when ρ is close to zero in Scenario B, the manufacturer's inventory may experience near zero variability. Setting $T_i = 1$ in (30) provides the manufacturer's inventory variance under the OUT/MMSE strategy,

$$\sigma_{MNS_{A,1}}^2 = \frac{\sigma_\varepsilon^2}{(1-\rho)^2} \left(T_s + \frac{2\rho^{3+T_p+T_s}}{1-\rho} + \frac{1-\rho^2 \left(1 + \rho^{T_p} (2 + 2\rho - \rho^{2+T_p} + \rho^{2(2+T_s)+T_p}) \right)}{(1-\rho)(1+\rho)} \right). \quad (32)$$

5.2. Bullwhip behavior in the three scenarios

The retailer's order variance generated in Scenario A and B are identical as

$$\sigma_{O_A}^2 = \sigma_{O_B}^2 = \sigma_O^2 = \frac{\sigma_\varepsilon^2}{(1-\rho)^2} \left(\frac{2 \left(\frac{T_i-1}{T_i} - \rho \right)^2 \rho^{2(T_p+1)} + \frac{2(1+\rho)}{T_i} \left(\frac{T_i-1}{T_i} - \rho \right) \rho^{T_p+1}}{(1+\rho) \left(\frac{2T_i-1}{T_i} \right) \left(1 - \rho \left(\frac{T_i-1}{T_i} \right) \right)} + \frac{1}{2T_i-1} \right). \quad (33)$$

$\sigma_{O_{A,1}}^2$ can be obtained as a particular case of (33) with $T_i = 1$. When $\rho = 0$, $\sigma_O^2 = \sigma_D^2 / (2T_i - 1)$, and is independent of T_p . Here the order variance is; decreasing in T_i , zero when $T_i = \infty$,

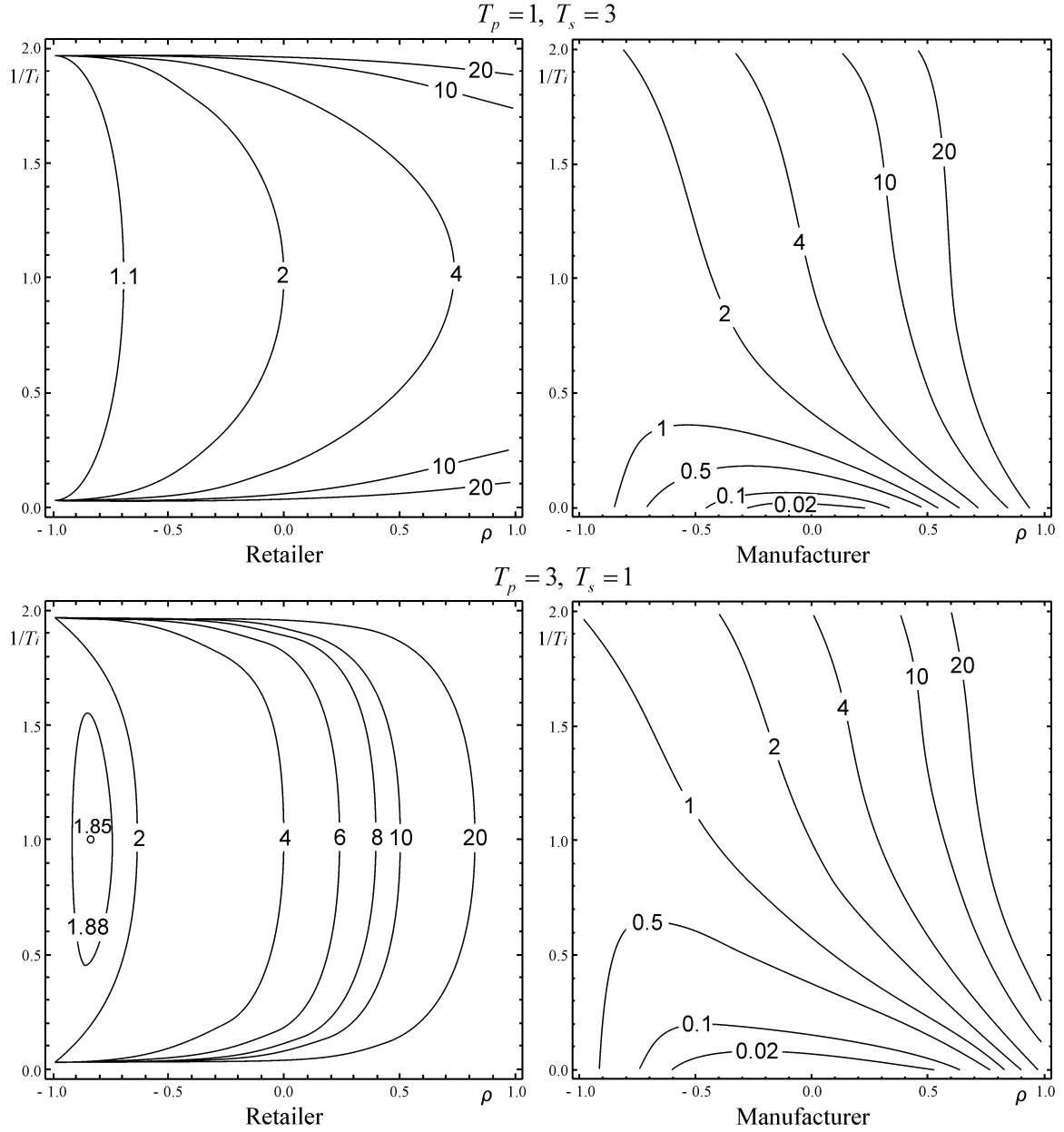


Figure 4. The net stock variance in a two-echelon supply chain for Scenario B when $\sigma_\varepsilon^2 = 1$

equal to the demand variance when $T_i = 1$, and approaches infinity when $T_i \rightarrow 0.5$.

For $\rho > 0$, and any T_p , when $T_i^{-1} < 1 - \rho$, $\sigma_o^2 < \sigma_D^2$, see Appendix IV. σ_o^2 is decreasing in T_i and equal to σ_D^2 when $T_i^{-1} = 1 - \rho$. For $\rho > 0$, (33) is a decreasing function of T_p when $T_i^{-1} < 1 - \rho$. When $T_i^{-1} > 1 - \rho$, (33) increases in T_p .

For $\rho < 0$, T_p has an odd-even lead-time effect on the retailer's order variance. When

T_p increases, the order variance oscillates but converges to $\sigma_o^2|_{T_p \rightarrow \infty} = \sigma_\varepsilon^2 / ((2T_i - 1)(1 - \rho)^2)$.

If $T_i > 1$, it is possible to eliminate the bullwhip effect. If $T_i = 1$, the OUT policy is present and $\sigma_o^2 > \sigma_D^2$. While not generally recommended, $0.5 < T_i < 1$ is stable and sometimes allows bullwhip to be avoided when demand is negatively correlated.

The manufacturer's order variance in the POUT/MMSE strategy is

$$\sigma_{P_A}^2 = \frac{\sigma_\varepsilon^2}{(1 - \rho)^2} \left(\frac{2\rho^{2(3+T_p+T_s)}}{1 + \rho} + \frac{\frac{1}{T_i} + \frac{2}{T_i} \left(\frac{T_i - 1}{T_i} \right) \rho^{1+T_p} + 2 \left(\left(\frac{T_i - 1}{T_i} \right) \rho^{1+T_p} \right)^2}{1 + \frac{T_i - 1}{T_i}} - \frac{2\rho^{3+T_p+T_s} \left(\frac{1}{T_i^2} + \left(\frac{T_i - 1}{T_i} \right) \rho^{1+T_p} \left(1 + \frac{1}{T_i} - \rho \right) \right)}{1 - \left(\frac{T_i - 1}{T_i} \right) \rho} \right). \quad (34)$$

The effect of the lead times disappears when $\rho = 0$ as (34) reduces to $\sigma_{P_A}^2 = \sigma_{O_A}^2 = \sigma_\varepsilon^2 / (2T_i - 1)$. The manufacturer's order variance under the OUT/MMSE policy can be obtained by setting $T_i = 1$ in (34):

$$\sigma_{P_{A,1}}^2 = \frac{\sigma_\varepsilon^2}{(1 - \rho)^2} \left(\frac{2\rho^{2(3+T_p+T_s)}}{1 + \rho} + 1 - 2\rho^{3+T_p+T_s} \right). \quad (35)$$

The manufacturer's order variance under the POUT/PFG strategy is

$$\sigma_{P_B}^2 = \frac{\sigma_\varepsilon^2}{(1 - \rho)^2} \left(1 + \frac{2 \left(\frac{T_i - 1}{T_i} \right)^{2T_s} (1 - T_i)^4 (1 - \rho^{1+T_p})^2}{T_i^3 (2T_i - 1)} - \frac{2 \left(\frac{T_i - 1}{T_i} \right)^{T_s} (1 - T_i)^2 (1 - \rho^{1+T_p})}{T_i^2} + \frac{2\rho^{2(3+T_p+T_s)}}{1 + \rho} - 2\rho^{3+T_p+T_s} \left(1 - \frac{\left(\frac{T_i - 1}{T_i} \right)^{T_s} (1 - T_i)^2 (1 - \rho^{1+T_p}) (1 + T_i (1 - \rho))}{T_i^2 (\rho + T_i (1 - \rho))} \right) \right). \quad (36)$$

(36) shows that both T_p and T_s have odd-even exponents which create oscillations in $\sigma_{P_B}^2$

when $\rho < 0$. When $T_i^{-1} = 1 - \rho$, the impact of T_p disappears as (36) reduces to

$$\sigma_{P_B}^2 = \sigma_\varepsilon^2 \left(\frac{1 + \rho - 2\rho^{2+T_s} - 2\rho^{3+T_s} + 2\rho^{4+2T_s}}{(1-\rho)^2(1+\rho)} \right). \quad (37)$$

(37) is: an increasing function of T_s ; always greater than σ_D^2 when $\rho > 0$; equal to σ_D^2 when $\rho = 0$ regardless of T_p and T_s ; and when $\rho < 0$, $\sigma_{P_B}^2$ oscillates around, and converges to σ_D^2 as T_s increases.

For $T_i^{-1} < 1 - \rho$, when $\rho = 0$, the effect of T_p disappears in (36). $\sigma_{P_B}^2$ is increasing in T_s and has a limit of σ_D^2 . When $\rho > 0$, $\sigma_{P_B}^2$ is increasing in T_s and decreasing in T_p , and $\sigma_{P_B}^2 < \sigma_D^2$ is possible when T_s is small and T_i is large. When $\rho < 0$, the bullwhip behavior is rather complex, but it can be shown that $\sigma_{P_B}^2$ is increasing in T_s and we can weaken the odd-even effect of T_p by increasing T_i .

For $T_i^{-1} > 1 - \rho$, when $\rho = 0$, T_s has an odd-even lead-time effect on $\sigma_{P_B}^2$ which oscillates around and converges to σ_D^2 . For positive ρ , and $(1 - \rho) < T_i^{-1} < 1$, increasing either lead-time, increases the order variance, and $\sigma_{P_B}^2 > \sigma_D^2$. When $T_i < 1$ or $\rho < 0$, it is possible to avoid generating bullwhip although the relationship between T_p , T_s , T_i and the bullwhip effect is rather complicated. As $T_i \rightarrow \infty$, $\sigma_{P_A}^2 \rightarrow 0$ and

$$\sigma_{P_B}^2 \rightarrow \frac{\rho^{2(1+T_p)}(1 + \rho + 2\rho^{2(2+T_s)} - 2\rho^{2+T_s}(1 + \rho))}{(\rho - 1)^2(1 + \rho)}.$$

Together with (35), these facts mean that there always exist a T_i such that $\sigma_{P_A}^2 < \sigma_{P_{A,1}}^2$ and $\sigma_{P_A}^2 < \sigma_{P_B}^2$.

5.3. Inventory and capacity cost analysis

Until this point in our variance and nervousness analysis, we have not made any specific assumptions about the distribution of the error term, ε_i . However, to be able to conduct an economic analysis we now need to assume ε_i is normally distributed. Then, as a

linear system exists, all the system states will also be normally distributed, and we can characterize performance analytically⁴.

Let $H(B)$ be the cost of holding (backlogging) one unit of inventory for one period. The inventory cost function at an individual echelon is $J_{NS,t} = H(NS_t)^+ + B(-NS_t)^+$, and the optimal target net stock level, $TNS^* = \sigma_{NS} z$ with $z = \Phi^{-1} \left[B(B+H)^{-1} \right]$ ensures that $B(B+H)^{-1} \times 100\%$ of periods end with inventory in stock (Brown 1962). Here $\Phi^{-1}[\cdot]$ is the inverse cumulative distribution function of the standard normal distribution. The expected, per period, inventory cost at each echelon is then

$$J_{NS} = E[J_{NS,t}] = \sigma_{NS} (B+H) \varphi[z], \quad (38)$$

where $\varphi[\cdot]$ is the probability density function of the standard normal distribution. Note (38) can be readily adapted for the inventory cost at the retailer (denoted J_{RNS}) and the manufacturer (indicated by J_{MNS}) by using the standard deviation of the appropriate net stock levels.

Corollary 1: When $T_i > 1$, the relationship $J_{MNS_B} < J_{MNS_A} < J_{MNS_{A,1}}$ holds for all ρ , T_p and T_s .

Proof: This follows directly from (38), inventory costs are linear functions of σ_{NS} , and $\sigma_{NS_B}^2 < \sigma_{NS_A}^2 < \sigma_{NS_{A,1}}^2$ holds. \square

When the retailer uses the POUT/PFG strategy, larger ρ lead to greater reductions in the manufacturer's inventory because of the linear relationship between the standard

⁴ Without this normality assumption, we would have to resort to a simulation based analysis as the required convolution of the pdf's involved quickly becomes intractable.

deviation of the inventory levels and inventory costs. Larger T_i leads to greater reductions in inventory cost at the manufacturer and reduces nervousness.

To allocate capacity costs to the production variability, we assume that each echelon works regular shifts, guaranteeing labor $\mu + s$ hours of work each week. Here μ is the average demand and s is an amount of space capacity above (or below) μ . When P_t corresponds to less work than the guaranteed hours, the workers receive a full weekly wage, despite standing idle for some of it. If P_t requires more hours of work than the guaranteed hours, flexible overtime is used to make up the difference. Let U represent the cost of producing one unit during regular working time and W represent the unit overtime cost, $W \geq U$. The capacity cost function to be minimized is $E[J_{P,t}]$, where $J_{P,t} = U(\mu + s) + W(P_t - \mu - s)^+$. Hosoda and Disney (2012) show that the optimal slack capacity, $s^* = \sigma_p z_p$ with $z_p = \Phi^{-1}[(W - U)/W]$, results in $(U/W) \times 100\%$ of periods using overtime. Then, under the optimal slack capacity, the manufacturer's capacity cost is

$$J_p = E[J_{P,t}] = \mu U + \sigma_p W \phi[z_p]. \quad (39)$$

Note the capacity cost (39) mechanism can also be applied to both the manufacturer and the retailer (after substituting in the appropriate standard deviation of the orders).

5.4. Numerical investigations

In this section, we will explore a numerical setting by assuming: the lead-times $T_p = T_s = 1$, the AR(1) demand process has an autocorrelation coefficient of $\rho = 0.4$ and mean $\mu = 12$, and the nervousness weight, $w = 0.5$. Table 2 highlights the economic consequences of different objectives at the retailer. Here, $B = 9$ and $H = 1$ at both the retailer and the manufacturer, implying an economic stockout probability at each echelon of 10%.

We also assume that labor receives 150% of the regular hourly wage for working overtime by using $U = 4$ and $W = 6$.

As J_O , J_P and J_{MNS} are decreasing in T_i , if the sole objective is to minimize J_O , J_P or J_{MNS} then $T_i^* = \infty$, $J_{RNS} = \infty$, $J_O = J_P = \mu U$ and $J_S \rightarrow \infty$. These objectives are denoted as an empty set \emptyset in Table 2. The bold numbers in Table 2 illustrate the OUT/MMSE policy and, as $T_i = 1$ for this strategy, these represent all possible cost functions. Furthermore, when the retailer is only interested in minimizing his inventory cost, both POUT strategies degenerate into the OUT strategy.

For a local cost optimizing retailer with both capacity and inventory costs, then $T_i^* > 1$ implying that one of the POUT strategies should be adopted. Although there is an increase in the retailer's inventory cost, the capacity cost in both echelons and the manufacturer's inventory cost and nervousness are reduced. Table 2 confirms Corollary 1, more accurate future guidance reduces the manufacturer's inventory cost. However, the PFG mechanism slightly increases the manufacturer's capacity cost. For all cost structures at the manufacturer, Scenario B requires a larger T_i than Scenario A. Then, $J_{MNS_B} < J_{MNS_A}$, $J_{P_B} > J_{P_A}$.

Table 2 shows that a local cost optimizing retailer cannot coordinate the whole supply chain. However, if the retailer can understand the structure of total supply chain costs and act altruistically, supply chain costs reduce. The manufacturer has the responsibility to trust the retailer's future guidance and to share the spoils of the retailer's selfless act. This altruistic behavior may be present if a single company owns both echelons. Table 2 also shows that both POUT strategies successfully reduce the total supply chain costs and nervousness compared to the OUT/MMSE approach. The POUT/MMSE strategy is the most economical but is more nervous than the POUT/PFG strategy.

A,1: <i>OUT / MMSE</i> A: <i>POUT / MMSE</i> B: <i>POUT / PFG</i>			Capacity cost objectives							
			J_O		J_P		$J_O + J_P$		\emptyset	
Inventory cost objectives	J_{RNS}	<i>Scenario</i>	<i>A</i>	<i>B</i>	<i>A</i>	<i>B</i>	<i>A</i>	<i>B</i>	(A,1), A, B	
		T_i^*	2.5	2.5	2.59	3.36	4.08	5.18	1	
		J_{RNS}	3.54	3.54	3.57	3.87	4.14	4.51	3.02	
		J_O	49.9	49.9	49.86	49.63	49.48	49.32	51.41	
		J_{MNS}	2.62	2.34	2.57	1.92	2.09	1.46	3.95	
		J_P	50	50.96	49.96	50.58	49.57	50.07	51.6	
		J_S	106.05	106.73	105.97	106.01	105.27	105.37	109.98	
		Δ	0.72	0.61	0.69	0.41	0.43	0.24	2.44	
	J_{MNS}	\emptyset								
	$J_{RNS} + J_{MNS}$	<i>Scenario</i>	<i>A</i>	<i>B</i>	<i>A</i>	<i>B</i>	<i>A</i>	<i>B</i>	<i>A</i>	<i>B</i>
		T_i^*	4.57	4.76	4.61	5.82	6.14	7.19	2.89	3.34
		J_{RNS}	4.31	4.37	4.32	4.72	4.82	5.14	3.69	3.86
		J_O	49.4	49.38	49.4	49.25	49.22	49.14	49.76	49.64
		J_{MNS}	1.98	1.54	1.97	1.36	1.73	1.2	2.45	1.93
		J_P	49.48	50.16	49.48	49.94	49.3	49.74	49.86	50.59
		J_S	105.18	105.45	105.17	105.28	105.08	105.22	105.76	106.02
	Δ	0.38	0.27	0.42	0.21	0.29	0.16	0.61	0.41	
	\emptyset	\emptyset								

Table 2. Numerical solutions for $T_p = T_s = 1$, $\rho = 0.4$, $\mu = 12$, $w = 0.5$

The total supply chain inventory costs in both POUT policies are convex in T_i . It's hard to find the value of T_i^* analytically, but from extensive numerical investigations we propose:

Conjecture 1: For any feasible objective function other than solely minimizing the retailer's inventory cost, $J_{SNS_B} < J_{SNS_A} < J_{SNS_{A,1}}$ and $J_{SO_A} < J_{SO_B} < J_{SO_{A,1}}$ hold for all $\{T_p, T_s\}$ when $\rho \geq 0$.

This conjecture proposes that both the POUT strategies can reduce inventory and capacity costs in a supply chain, even if the retailer is not striving to minimize these objectives. The POUT/PFG approach can achieve the lowest supply chain inventory cost and

the least nervous system. However, the POUT/MMSE strategy has better capacity performance, so when the total supply chain inventory related cost is of concern an altruistic retailer should adopt the POUT/PFG policy. By optimizing the supply chain inventory cost, there will also be reductions in both nervousness and supply chain capacity costs. However, when the manufacturer's capacity cost is of primary importance, POUT/MMSE is recommended.

6. Managerial implications

We have analyzed an innovative adaptation to an MRP system, the POUT replenishment policy, and accounted for its future impact on the call-off orders via the PFG mechanism. These techniques are suitable for scheduling high volume items. The POUT policy is closely related to the OUT policy that is readily available native in many MRP systems. The POUT/PFG policy can be incorporated into an ERP system by custom-coding user defined macros.

An established MRP system is needed to implement these techniques. We have noticed in many cases MRP systems are not always: fully implemented, used consistently, or without data accuracy issues. Thus, there may be some work required to get an MRP ready for implementation of the POUT/PFG technique. Furthermore, in addition to the usual decisions regarding forecast method and parameter selection, an additional choice for the value of T_i is required. We have found that longer lead-times need, and higher volumes support, the use of larger values of T_i . In practice we have found a simulation of the POUT/PFG method, with real demand data and lead-times, in an off-line spreadsheet, is a good way to select T_i .

The POUT/PFG leads to a smoother call-off order and more accurate future order forecasts, not only reducing the variability but also increasing the predictability, of the

supplier's workload. Suppliers receive more stable demand reducing their need for inventory, burst capacity, and expedient transportation. The stable demand also makes it easier for suppliers to schedule maintenance. Better maintenance improves up-time and reduces costs. Over time, a better commercial relationship is created, resulting in better negotiations during contract renewal.

7. Concluding remarks

We have jointly considered the MRP nervousness and bullwhip problems which were previously treated as separate issues in the literature. We analyzed two methods of generating order call-offs and two methods of creating order forecasts. We developed a new measure of MRP nervousness based on a geometrically weighted sum of order forecast errors. While it is known that the POUT replenishment policy can reduce the order variability at the retailer, we revealed that it also reduces the manufacturer's MRP nervousness. This is consistent with the intuition that smoother orders are easier to forecast. Accounting for the future consequences of the POUT policy with the PFG mechanism further improves the accuracy of the future guidance. By tuning the feedback controller, T_i , it is always possible to avoid the bullwhip effect at both echelons.

Our analysis reveals that when the retailer adopts either of the POUT strategies, the manufacturer can reduce MRP nervousness and inventory costs, as well as reducing the capacity costs at each echelon. We reveal that the MRP nervousness, the inventory variance at the manufacturer and the order variance at both echelons are all commingled and affected by T_i which can be selected to minimize a range of cost functions.

Our analysis recommends the: OUT/MMSE policy when the retailer's inventory cost is of sole concern; POUT/MMSE policy when the manufacturer's capacity cost is the only concern; and the POUT/PFG policy when the supply chain is concerned with inventory costs

and MRP nervousness. Our proposed strategies are easy to understand and—since they do not require a change in the commercial relationship—are relatively easy to implement in practice.

The implications of Corollary 1 are that our POUT/PFG policy means the manufacturer requires less safety stock than the OUT/MMSE policy, Brown (1962). By Grasso and Taylor (1984), who demonstrated that safety stock is more efficient than safety lead-times at reducing nervousness, we conjecture that our POUT/PFG policy is also more effective than using safety lead-times to reduce nervousness. However, the comparison between our POUT/PFG policy and the frozen period and lot sizing solutions to the MRP nervousness is not practical as our model does not consider these factors. Future research could be directed towards these areas. The performance of the POUT/PFG mechanism in more realistic supply chain settings (for example in divergent supply chains, over more echelons, or with stochastic lead times) could be investigated. The impact of more realistic modeling assumptions (for example real demand data, capacity constraints, or promotions) could be studied. Here a simulation or system dynamics approach might be useful.

8. References

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Appendix I. List of nomenclature

P_t	Manufacturer's production order quantity at time t .
O_t	Retailer's order quantity at time t .
$\hat{O}_{t,t+j}$	j -period ahead forecast of the retailer's orders.
D_t	End customer demand at time t .
$\hat{D}_{t,t+j}$	Demand forecast, made at time t , of demand in period $t+j$.
ρ	Auto-regressive demand correlation
NS_t	Net stock at time t .
RNS_t	Retailer's net stock at time t .
MNS_t	Manufacturer's net stock at time t .
T_p	Retailer's lead-time.
T_s	Manufacturer's lead-time.
T_i	Proportional feedback controller.
T_i^*	Optimal proportional feedback controller.
Δ_A	Nervousness in the POUT/MMSE strategy.
$\Delta_{A,1}$	Nervousness in the OUT/MMSE strategy.
Δ_B	Nervousness in the POUT/PFG strategy.
$\Delta[j]$	The variance of the j -step ahead order forecast error.
w	A weighting factor that determines how quickly the variance of the future order forecast error decays away in the nervousness measure.
J_O	Retailer's capacity cost.
J_{RNS}	Retailer's inventory cost.
J_P	Manufacturer's capacity cost.
J_{MNS}	Manufacturer's inventory cost.
J_{SO}	Supply chain capacity related cost $J_{SO} = J_O + J_P$.
J_{SNS}	Supply chain inventory related cost $J_{SNS} = J_{RNS} + J_{MNS}$.
J_S	Total supply chain cost $J_S = J_{SO} + J_{SNS}$.

Appendix II. Measuring the accuracy of the future order stream

The call-off order in (1) generated by the POUT policy is

$$O_t = \left(1 - \frac{1}{T_i}\right) O_{t-1} + \left(\hat{D}_{t,t+T_p+1} - \hat{D}_{t-1,t+T_p}\right) + \left(\sum_{i=1}^{T_p} \hat{D}_{t,t+i} - \sum_{i=1}^{T_p} \hat{D}_{t-1,t+i-1} + D_t\right) / T_i. \quad (A1)$$

Letting $\mu = 0$ in (6), then the AR(1) demand becomes

$$D_t = \rho D_{t-1} + \varepsilon_t = \sum_{i=0}^{\infty} \rho^i \varepsilon_{t-i} = \varepsilon_t + \rho \varepsilon_{t-1} + \rho^2 \varepsilon_{t-2} + \dots \quad (A2)$$

Using (8), (9) and (A2) in (A1), O_t can be written as

$$O_t = \left(1 - \frac{1}{T_i}\right) O_{t-1} + \left(\rho^{T_p+1} + \frac{\kappa}{T_i}\right) \varepsilon_t + \rho^{T_p+1} \left(\rho - 1 + \frac{1}{T_i}\right) D_{t-1}; \quad D_{t-1} = \sum_{i=0}^{\infty} \rho^i \varepsilon_{t-i-1} \quad (A3)$$

where $\kappa = (1 - \rho^{T_p+1}) / (1 - \rho)$. Substituting (A3) into itself recursively yields

$$O_t = \sum_{n=0}^{\infty} \left(\left(\frac{T_i-1}{T_i} \right)^n \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) + \rho^{T_p+1} \left(\rho^n - \left(\frac{T_i-1}{T_i} \right)^n \right) \right) \varepsilon_{t-n}. \quad (A4)$$

The variance of O_t is derived by finding the expected value of the square of (A4),

$$\sigma_O^2 = \sigma_\varepsilon^2 \left(\frac{\kappa^2}{2T_i - 1} + \frac{2\kappa\rho^{1+T_p}}{\rho + T_i(1 - \rho)} - \frac{\rho^{2(1+T_p)}}{\rho^2 - 1} \right). \quad (A5)$$

When demand is i.i.d., the variance of the retailer's orders reduces to $\sigma_O^2 = \sigma_\varepsilon^2 / (2T_i - 1)$.

In Scenario A, the order forecast error is

$$\begin{aligned} \hat{D}_{t-j,t+T_p+1} - O_t &= \sum_{n=j}^{\infty} \rho^{T_p+1+n} \varepsilon_{t-n} - \sum_{n=0}^{\infty} \left(\left(\frac{T_i-1}{T_i} \right)^n \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) + \rho^{T_p+1} \left(\rho^n - \left(\frac{T_i-1}{T_i} \right)^n \right) \right) \varepsilon_{t-n} \\ &= \left(\sum_{n=0}^{j-1} \left(- \left(\frac{T_i-1}{T_i} \right)^n \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) - \rho^{T_p+1} \left(\rho^n - \left(\frac{T_i-1}{T_i} \right)^n \right) \right) \varepsilon_{t-n} + \right. \\ &\quad \left. \sum_{n=j}^{\infty} \left(\rho^{T_p+1+n} - \left(\frac{T_i-1}{T_i} \right)^n \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) - \rho^{T_p+1} \left(\rho^n - \left(\frac{T_i-1}{T_i} \right)^n \right) \right) \varepsilon_{t-n} \right). \end{aligned} \quad (A6)$$

$\Delta_A[j]$, the variance of the j period ahead order forecast error in Scenario A is

$$\begin{aligned} \Delta_A[j] &= \sigma_\varepsilon^2 \left(\sum_{n=0}^{j-1} \left(\left(\frac{T_i-1}{T_i} \right)^n \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) + \rho^{T_p+1} \left(\rho^n - \left(\frac{T_i-1}{T_i} \right)^n \right) \right)^2 + \right. \\ &\quad \left. \sum_{n=j}^{\infty} \left(\rho^{T_p+1+n} - \left(\frac{T_i-1}{T_i} \right)^n \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) - \rho^{T_p+1} \left(\rho^n - \left(\frac{T_i-1}{T_i} \right)^n \right) \right)^2 \right) \\ &= \sigma_\varepsilon^2 \left(\frac{\kappa^2}{T_i^2 \left(1 - \left(\frac{T_i-1}{T_i} \right)^2 \right)} + \frac{2\kappa\rho^{1+T_p} \left(1 - \left(\frac{\rho(T_i-1)}{T_i} \right)^j \right)}{T_i \left(1 - \left(\frac{\rho(T_i-1)}{T_i} \right) \right)} + \frac{\rho^{2(1+T_p)} (1 - \rho^{2j})}{1 - \rho^2} \right), \end{aligned} \quad (A7)$$

and the nervousness of Scenario A is

$$\Delta_A = \sigma_\varepsilon^2 \sum_{j=1}^{\infty} w(1-w)^{j-1} \left(\sum_{n=0}^{j-1} \left(\left(\frac{T_i-1}{T_i} \right)^n \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) + \rho^{T_p+1} \left(\rho^n - \left(\frac{T_i-1}{T_i} \right)^n \right) \right)^2 + \sum_{n=j}^{\infty} \left(\rho^{T_p+1+n} - \left(\frac{T_i-1}{T_i} \right)^n \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) - \rho^{T_p+1} \left(\rho^n - \left(\frac{T_i-1}{T_i} \right)^n \right) \right)^2 \right) \quad (A8)$$

$$= \sigma_\varepsilon^2 \left(\frac{\kappa^2}{2T_i-1} + \frac{2\kappa\rho^{T_p+1}}{T_i-(T_i-1)(1-w)\rho} + \frac{\rho^{2T_p+2}}{1-(1-w)\rho^2} \right).$$

For Scenario B, we may rewrite the future forecasted order as

$$\begin{aligned} \hat{O}_{t,t+j} &= \frac{1}{T_i} \left(\frac{T_i-1}{T_i} \right)^j \left(\sum_{i=1}^{T_p} \hat{D}_{t,t+i} - \left(\sum_{i=1}^{T_p} O_{t-i} + RNS_t \right) \right) + \hat{D}_{t,t+T_p+1+j} + \frac{1}{T_i} \left(\frac{T_i-1}{T_i} \right)^j TNS \\ &= \left(\frac{T_i-1}{T_i} \right)^{j+1} O_{t-1} + \left(\hat{D}_{t,t+T_p+1+j} - \left(\frac{T_i-1}{T_i} \right)^j \hat{D}_{t-1,t+T_p} \right) \\ &\quad + \frac{1}{T_i} \left(\frac{T_i-1}{T_i} \right)^j \left(\sum_{i=1}^{T_p} \hat{D}_{t,t+i} - \sum_{i=1}^{T_p} \hat{D}_{t-1,t+i-1} \right) + \frac{1}{T_i} \left(\frac{T_i-1}{T_i} \right)^j D_t. \end{aligned} \quad (A9)$$

Assuming $TNS = 0$, substituting (8), (9) and (A2) into (A9) yields

$$\hat{O}_{t,t+j} = \left(\frac{T_i-1}{T_i} \right)^{j+1} O_{t-1} + \left(\rho^{T_p+1+j} + \frac{\kappa}{T_i} \left(\frac{T_i-1}{T_i} \right)^j \right) \varepsilon_t + \rho^{T_p+1} \left(\rho^{j+1} - \left(\frac{T_i-1}{T_i} \right)^{j+1} \right) D_{t-1}. \quad (A10)$$

Both O_t and $\hat{O}_{t-j,t}$ can be written as $\phi O_{t-j-1} + \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots$, the form of an

ARMA($j-1, \infty$) process. The coefficients in $\hat{O}_{t-j,t}$ and O_t are summarized in [Table A3](#).

Coefficient	$\hat{O}_{t-j,t}$	O_t
ϕ	$\left(\frac{T_i-1}{T_i} \right)^{j+1}$	$\left(\frac{T_i-1}{T_i} \right)^{j+1}$
θ_0	0	$\rho^{T_p+1} + \frac{\kappa}{T_i}$
θ_1	0	$\left(\frac{T_i-1}{T_i} \right) \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) + \rho^{T_p+1} \left(\rho - 1 + \frac{1}{T_i} \right)$
θ_2	0	$\left(\frac{T_i-1}{T_i} \right)^2 \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) + \rho^{T_p+1} \left(\rho^2 - \left(\frac{T_i-1}{T_i} \right)^2 \right)$
θ_n	0	$\left(\frac{T_i-1}{T_i} \right)^n \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) + \rho^{T_p+1} \left(\rho^n - \left(\frac{T_i-1}{T_i} \right)^n \right)$
θ_j	$\rho^{T_p+1+j} + \left(\frac{T_i-1}{T_i} \right)^j \frac{\kappa}{T_i}$	$\left(\frac{T_i-1}{T_i} \right)^j \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) + \rho^{T_p+1} \left(\rho^j - \left(\frac{T_i-1}{T_i} \right)^j \right)$
θ_{j+1}	$\rho^{T_p+1} \left(\rho^{j+1} - \left(\frac{T_i-1}{T_i} \right)^{j+1} \right)$	$\rho^{T_p+1} \left(\rho^{j+1} - \left(\frac{T_i-1}{T_i} \right)^{j+1} \right)$

θ_{j+2}	$\rho^{T_p+1} \left(\rho^{j+1} - \left(\frac{T_i-1}{T_i} \right)^{j+1} \right) \rho$	$\rho^{T_p+1} \left(\rho^{j+1} - \left(\frac{T_i-1}{T_i} \right)^{j+1} \right) \rho$
θ_{j+n}	$\rho^{T_p+1} \left(\rho^{j+1} - \left(\frac{T_i-1}{T_i} \right)^{j+1} \right) \rho^{n-1}$	$\rho^{T_p+1} \left(\rho^{j+1} - \left(\frac{T_i-1}{T_i} \right)^{j+1} \right) \rho^{n-1}$

Table A3. The coefficients of $\hat{O}_{t-j,t}$

The coefficients θ_i for $i \geq j$ in $\hat{O}_{t-j,t}$ and O_t are identical. Note that the order error can be converted into an MA($j-1$) process where only $\varepsilon_t, \varepsilon_{t-1}, \dots, \varepsilon_{t-j+1}$ have an influence on the forecasted order error. The forecast error in Scenario B is

$$\hat{O}_{t-j,t} - O_t = \sum_{n=0}^{j-1} \left(-\left(\frac{T_i-1}{T_i} \right)^n \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) - \rho^{T_p+1} \left(\rho^n - \left(\frac{T_i-1}{T_i} \right)^n \right) \right) \varepsilon_{t-n}. \quad (A11)$$

Then $\Delta_B[j]$, can be obtained by taking the expectation of (A11)

$$\begin{aligned} \Delta_B[j] &= \sigma_\varepsilon^2 \sum_{n=0}^{j-1} \left(\left(\frac{T_i-1}{T_i} \right)^n \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) + \rho^{T_p+1} \left(\rho^n - \left(\frac{T_i-1}{T_i} \right)^n \right) \right)^2 \\ &= \sigma_\varepsilon^2 \left(\frac{\left(\left(\frac{T_i-1}{T_i} \right)^{2j} - 1 \right) \kappa^2}{\left(\left(\frac{T_i-1}{T_i} \right)^2 - 1 \right) T_i^2} + \frac{\rho^{2+2T_p} (1 - \rho^{2j})}{1 - \rho^2} + \frac{2\kappa \rho^{1+T_p} \left(\left(\rho \left(\frac{T_i-1}{T_i} \right) \right)^j - 1 \right)}{T_i \left(\rho \left(\frac{T_i-1}{T_i} \right) - 1 \right)} \right), \end{aligned} \quad (A12)$$

which is increasing in j . Finally, the accuracy of the POUT/PFG future order stream is given by

$$\begin{aligned} \Delta_B &= \sigma_\varepsilon^2 \sum_{j=1}^{\infty} w(1-w)^{j-1} \sum_{n=0}^{j-1} \left(\left(\frac{T_i-1}{T_i} \right)^n \left(\rho^{T_p+1} + \frac{\kappa}{T_i} \right) + \rho^{T_p+1} \left(\rho^n - \left(\frac{T_i-1}{T_i} \right)^n \right) \right)^2 \\ &= \sigma_\varepsilon^2 \left(\frac{\kappa^2}{2T_i - 1 + w(T_i - 1)^2} + \frac{2\kappa \rho^{T_p+1}}{T_i - (T_i - 1)(1-w)\rho} + \frac{\rho^{2T_p+2}}{1 - (1-w)\rho^2} \right). \end{aligned} \quad (A13)$$

Appendix III. The variance of the manufacturer's net stock

Combining (27) and (28), we have

$$MNS_t = MNS_{t-1} + \sum_{j=1}^{T_s+1} \left(\hat{O}_{t-T_s-1, t-T_s-1+j} - \hat{O}_{t-T_s-2, t-T_s-2+j} \right) + O_{t-T_s-1} - O_t. \quad (A14)$$

From (A4) and the information in [Table A3](#), we may also obtain

$$\hat{O}_{t,t+j} = \sum_{n=0}^{\infty} \left(\rho^{T_p+1+j+n} + \frac{\kappa}{T_i} \left(\frac{T_i-1}{T_i} \right)^{j+n} \right) \varepsilon_{t-n}. \quad (\text{A15})$$

Using (A15), we can find

$$\hat{O}_{t-T_s-1,t-T_s-1+j} - \hat{O}_{t-T_s-2,t-T_s-2+j} = \left(\left(\rho^{T_p+1+j} + \frac{\kappa}{T_i} \left(\frac{T_i-1}{T_i} \right)^j \right) \varepsilon_{t-T_s-1} + \sum_{n=1}^{\infty} \left((\rho-1) \rho^{T_p+j+n} - \frac{\kappa}{T_i^2} \left(\frac{T_i-1}{T_i} \right)^{j-1+n} \right) \varepsilon_{t-T_s-1-n} \right). \quad (\text{A16})$$

Based on (A4), we calculate

$$O_{t-T_s-1} - O_t = \left(\sum_{n=0}^{T_s} \left(-\rho^{T_p+1+n} - \frac{\kappa}{T_i} \left(\frac{T_i-1}{T_i} \right)^n \right) \varepsilon_{t-n} + \sum_{n=0}^{\infty} \left(\frac{\kappa}{T_i} \left(\frac{T_i-1}{T_i} \right)^n \left(1 - \left(\frac{T_i-1}{T_i} \right)^{T_s+1} \right) + (1 - \rho^{T_s+1}) \rho^{T_p+1+n} \right) \varepsilon_{t-T_s-1-n} \right). \quad (\text{A17})$$

Using (A16) and (A17) in (A14), we can derive

$$MNS_t = MNS_{t-1} + \sum_{n=0}^{T_s} \left(-\rho^{T_p+1+n} - \frac{\kappa}{T_i} \left(\frac{T_i-1}{T_i} \right)^n \right) \varepsilon_{t-n} + \left(\kappa \left(1 - \left(\frac{T_i-1}{T_i} \right)^{T_s+1} \right) + \rho^{T_p+1} \sum_{i=0}^{T_s} \rho^i \right) \varepsilon_{t-T_s-1}. \quad (\text{A18})$$

Substituting (A18) into itself recursively shows the net stock series is an MA process

$$MNS_{t,B} = \sum_{n=0}^{T_s} \sum_{i=0}^n \left(-\rho^{T_p+1+i} - \frac{\kappa}{T_i} \left(\frac{T_i-1}{T_i} \right)^i \right) \varepsilon_{t-n} \quad (\text{A19})$$

and the variance of net stock level in the manufacturer for the POUT/PGF scenario becomes

$$\sigma_{MNS_B}^2 = \sigma_{\varepsilon}^2 \sum_{n=0}^{T_s} \left(\sum_{i=0}^n \left(\rho^{T_p+1+i} + \frac{\kappa}{T_i} \left(\frac{T_i-1}{T_i} \right)^i \right) \right)^2. \quad (\text{A20})$$

Following the same procedure, we can transform the POUT/MMSE net stock time series into

$$MNS_{t,A} = \sum_{n=0}^{T_s} \sum_{i=0}^n \left(-\rho^{T_p+1+i} - \frac{\kappa}{T_i} \left(\frac{T_i-1}{T_i} \right)^i \right) \varepsilon_{t-n} + \sum_{n=0}^{\infty} \left(-\left(\frac{T_i-1}{T_i} \right)^{1+n} \left(1 - \left(\frac{T_i-1}{T_i} \right)^{T_s+1} \right) \kappa \right) \varepsilon_{t-T_s-1-n}. \quad (\text{A21})$$

For the POUT/MMSE strategy, the manufacturer's net stock variance is

$$\sigma_{MNS_A}^2 = \sigma_{\varepsilon}^2 \left(\sum_{n=0}^{T_s} \left(\sum_{i=0}^n \left(\rho^{T_p+1+i} + \frac{\kappa}{T_i} \left(\frac{T_i-1}{T_i} \right)^i \right) \right)^2 + \sum_{n=0}^{\infty} \left(\left(\frac{T_i-1}{T_i} \right)^{1+n} \left(1 - \left(\frac{T_i-1}{T_i} \right)^{T_s+1} \right) \kappa \right)^2 \right). \quad (\text{A22})$$

It is clear to see from (A20) and (A22) that $\sigma_{MNS_B}^2 < \sigma_{MNS_A}^2$. The net stock variance maintained by the OUT/MMSE policy can be derived from (A22) by setting $T_i = 1$,

$$\sigma_{MNS_{A,1}}^2 = \sigma_\varepsilon^2 \left(\frac{T_s(\rho-1) - 2\rho^{3+T_p+T_s}}{(\rho-1)^3} + \frac{\rho^2(1 + \rho^{T_p}(2 + 2\rho - \rho^{2+T_p} + \rho^{2(2+T_s)+T_p})) - 1}{(\rho-1)^3(1+\rho)} \right). \quad (\text{A23})$$

Appendix IV. Proof that $T_i^{-1} \leq 1 - \rho$ enables $\sigma_o^2 \leq \sigma_D^2$ for $0 < \rho < 1$ and $T_p \in \mathbb{N}^0$

From (33), if $T_i^{-1} = 1 - \rho$, the effect of T_p disappears as (33) reduces to

$$\sigma_o^2 = \sigma_\varepsilon^2 / (1 - \rho^2), \quad (\text{A24})$$

which is the same as the demand variance, see (7). Thus, $\sigma_o^2 = \sigma_D^2$ for any demand correlation ρ and for all lead-times T_p .

When $T_i^{-1} < 1 - \rho$ and $0 < \rho < 1$, it is easy to ascertain from (33) that the retailer's order variance is decreasing in T_p . As $T_p \in \mathbb{N}^0$ the order variance at $T_p = 0$ is a maximum.

When $T_p = 0$, (33) simplifies to

$$\sigma_o^2|_{T_p=0} = \sigma_D^2 \frac{T_i(1 + \rho(3 - 2(1 - T_i(1 - \rho))\rho)) - \rho}{(2T_i - 1)(\rho + T_i(1 - \rho))}. \quad (\text{A25})$$

As $0.5 < T_i < \infty$ is required for stability, (A25) is always less than the demand variance when $T_i^{-1} < 1 - \rho$ for positive ρ .